JOURNAL OF MATERIALS SCIENCE 32 (1997) 1099–1103

Measurement of the dynamic elastic moduli of porous titanium aluminide compacts

T. E. MATIKAS, P. KARPUR, S. SHAMASUNDAR*
*Wright Laboratory, Materials Directorate, WL/MLLP/UDRI, and * Materials Directorate, WL/MLLN/UES, Wright Patterson Air Force Base, OH, 45433-7817 USA*

The dynamic elastic moduli of the porous alpha-two titanium aluminide compacts are measured using an ultrasonic technique. Both shear and longitudinal velocities are measured for compacts of different densities, making computation of all the four elastic constants, namely, the Young's modulus, shear modulus, bulk modulus and Poisson's ratio. The dependence of these on the relative density are correlated and compared with some earlier models, and some of the uncertainties in the earlier models are discussed.

1. Introduction

There is a significant amount of research directed towards the measurement as well as the predictive modeling of the effective macroscopic properties of heterogeneous multiphase media. Such information is important from engineering, technological and scientific points of view. The effective property models developed for multiphase materials have been successfully applied to porous materials by treating the porosity either as a randomly distributed or interconnected second phase. A general formulation for the effective property of a porous medium of the form:

$$\pi = \pi_0 f(\rho) \tag{1}$$

is popular in the literature, where π is the effective property (such as conductivity, strength, elastic moduli etc., π_0 is the corresponding property of the fully dense material, ρ is the relative density (equal to 1-p, where p is the porosity), and $f(\rho)$ is a function that correlates the property to relative density. If Young's modulus, for example, is the property of interest, several semi-empirical and analytical formulations can be found in the literature [1–5]. Some of these are listed below:

Duckworth [1]:

$$E(\rho) = E_0 \exp(-bp) \tag{2}$$

Phani and Niyogi [4] :

$$E(\rho) = E_0(1 - ap)^n \tag{3}$$

Ramakrishnan and Arunachalam [5]:

$$E(\rho) = E_0(1 - p^2)/(1 + b_0 p) \tag{4}$$

In Equations 2–4, various constants a, b, b_0, have been empirically determined. The models cited above and several other models available in literature have been tested for different materials [6, 7]. However, an understanding of the physical basis of various formulations; correlating the formulations to the specifications of the precursor powder material (i.e., the particle shape, size, distribution etc.,) and an understanding of the physical meaning of the constants used in the models, are still elusive. There are attempts to develop rigorous effective property models as well as to correlate the cross property relations. For example, the work of Milton [8] and Gibiansky and Torquato [9] relates the thermal conductivity to elastic moduli of composite materials. Such models although rigorous have not been validated with adequate experimental data.

The present paper reports the measurement of dynamic elastic moduli of porous compacts of a model material, alpha-two titanium aluminide (Ti–24Al–11Nb, at %) using an ultrasonic technique. The ultrasonic waves, which are elastic stresses of small amplitude, propagate with different modalities (compressional or shear) in the porous solid material. These elastic stress waves satisfy the basic equations of linear elasticity (Hooke's law) and their velocities of propagation in the material are directly related to the elastic moduli (Young's and shear) of the material at zero load (dynamic moduli). In principle, for a linear solid material, the dynamic modulus should be equal in value to the elastic modulus measured by any conventional mechanical test. However, in reality, because of the compliance of the testing machines etc., there is a difference in the values measured by the two methods. The dynamic moduli are generally (slightly) higher than the correspondent static moduli measured with a mechanical testing machine. For a linearly elastic material the dynamic modulus more accurately represents the actual modulus (i.e., the incremental resistance to strain) of the material.

The purpose of this study was to generate useful elastic property–relative density correlations for the model material used and to verify the available models in the light of the experimental data. The correlations developed in the present paper are of practical use to map the nonuniform distribution of elasto–

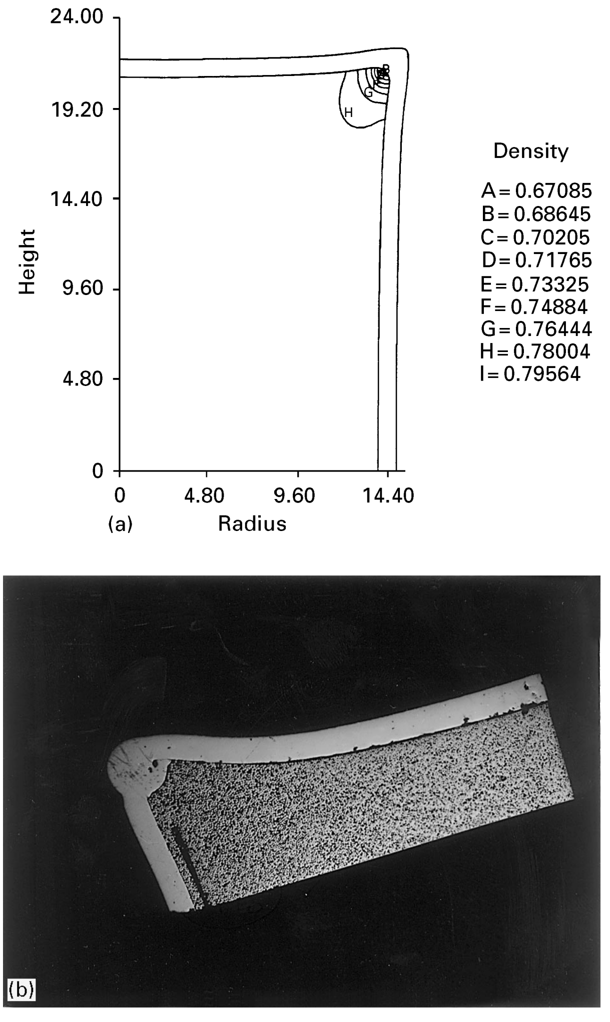

Figure 1 Non-uniform densification due to the 'can shielding' effect observed during hot isostatic pressing (HIPing) of alpha-two titanium aluminide in (a) finite element simulation model and (b) experiments.

mechanical properties in components produced by powder metallurgical processing. Fig. 1, for example shows the finite element simulation prediction and experimentally observed nonuniform density distribution in hot isostatic pressing of alpha-two titanium aluminide. The nonuniform density distribution which arises as a result of the interface friction (between powder and encapsulation), stiffness of the encapsulation material (in case of not isostatic pressing (HIPing)) etc., can at best only be minimized. A significant contribution of this work is the measurement of both longitudinal and shear velocities in an elastic half space and the deduction of all the elastic moduli by suitably accounting for the Poisson's ratio effect. This is an improvement in the measurement technique used by other workers, for example Wang [10].

2. Theory of dynamic moduli measurement

Ultrasonic waves are stress waves propagating in materials by inducing small (in the order of 10^{-7} mm) elastic deformations of the material. As a result, the ultrasonic propagation characteristics are mainly dependent on the elastic properties of the materials. Therefore, the waves propagation equations which are based on linear elasticity theory can be used to calculate the dynamic moduli of a material, if the stress wave velocities and density are measured independently [11, 12].

If an elastic stress wave is propagating in an isotropic semi-infinite medium due to the application of a normal or a shear traction on the surface of the solid medium, the displacement vector due to vibration particles in the material is governed by the equation of motion. The equation of motion can then be obtained from the equilibrium equations and the constitutive equations between stress and strain, where the factor of proportionality is the elastic tensor. The elastic tensor for an isotropic medium with density ρ can be written as a function of the Lamé constants, λ and μ.

In the case of an elastic stress applying normal traction on the surface of the material, the displacement vector in the solid is parallel to the axis of wave propagation in the material, and the motion equation will contain the velocity of propagation of the longitudinal ultrasonic waves, C_L, which is given by the expression:

$$C_L = \left(\frac{\lambda + \mu}{\rho}\right)^{1/2} \tag{5}$$

In the case of an elastic stress applying shear traction on the surface of the material, the displacement vector in the solid is tangential to the axis of wave propagation in the material and the motion equation will contain the velocity of propagation of the shear ultrasonic waves, C_S, which is given by the expression:

$$C_S = \left(\frac{\mu}{\rho}\right)^{1/2} \tag{6}$$

Since the Lamé constants are related to the Young's modulus, E, the shear modulus, G (which is the same as μ), the bulk modulus, K, and the Poisson's ratio, ν, the elastic moduli can be expressed as a function of the density, the longitudinal velocity and the shear velocity of the material as follows:

$$E = 4\rho C_S^2 \left[\frac{\frac{3}{4} - \left(\frac{C_S}{C_L}\right)^2}{1 - \left(\frac{C_S}{C_L}\right)^2}\right] \tag{7}$$

$$G = \rho C_S^2 \tag{8}$$

$$K = \rho C_S^2 \left(\left(\frac{C_S}{C_L}\right)^2 - \frac{4}{3}\right) \tag{9}$$

$$\nu = \frac{\frac{1}{2} - \left(\frac{C_S}{C_L}\right)^2}{1 - \left(\frac{C_S}{C_L}\right)^2} \tag{10}$$

It is easily seen that Equations 7–10 provide a practical framework for the measurement of all the elastic moduli using ultrasonic technique.

3. Experimental procedure

Alpha-two titanium aluminide powder produced by PREP process with a particle size distribution of $-140/+400$ (i.e., between 37–105 μm) was hot isostatically pressed (HIPed) to different relative densities at 1000 °C, for 1 h at pressures of 2, 7 and 70 MPa. In order to get samples with intermediate densities cylinders machined out of these HIPed samples were hot pressed at 1000 °C. By a combination of HIPing and hot die-pressing, samples with different relative densities (defined as the ratio of the density of a partially dense compact to the theoretical density of a fully dense compact) in the range of 0.7–1.0 were obtained. The relative density was measured by direct method. The density of the fully dense material is measured to be 4.7 g per cc. The initial packing density (or tap density) of the powder was found to be 0.63. Disks of 12.5 mm nominal diameter and 10 mm thickness were prepared for ultrasonic measurement. Fig. 2 (a and b) shows the typical porous microstructure of some of the samples used.

The experimental setup for the measurement of the velocities consisted of piezoelectric transducers (with x-cut or y-cut crystals for longitudinal and shear wave generation, respectively) operating in pulse-echo mode [11]. The ultrasonic frequency used was in the range of 2.25 – 5.0 MHz so that the wavelength of ultrasound was much larger than the inherent porosity in the material ($c = \lambda f$, where c is either longitudinal or shear wave velocity, λ is the corresponding wavelength, and f is the frequency of ultrasound).

Measured values of longitudinal and shear velocities for compacts of different relative densities were used in Equations 7–10 to determine various elastic moduli. Fig. 3 shows the measured longitudinal and shear velocities as a function of relative density.

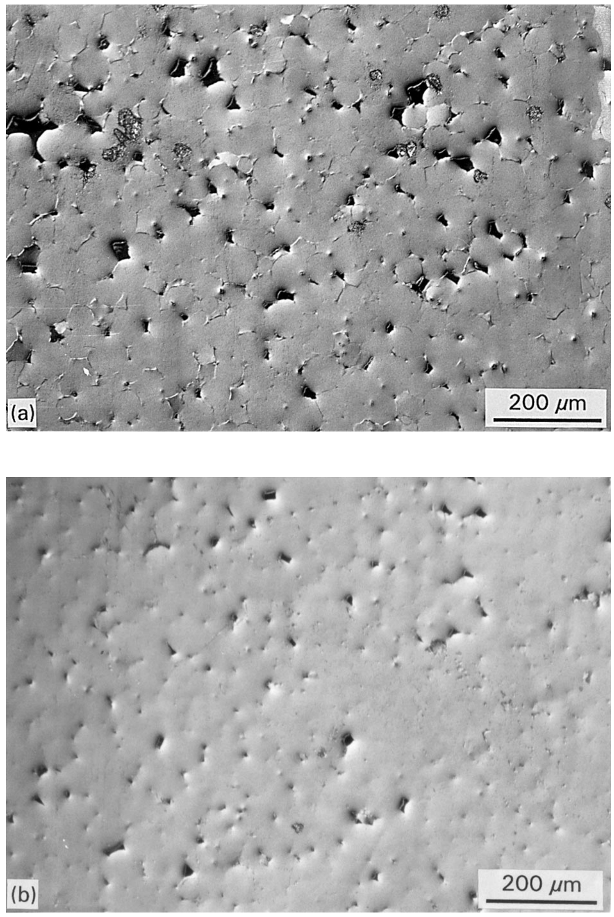

Figure 2 Porous microstructure of alpha-two titanium aluminide samples used in ultrasonic measurement. (a) HIPed at 2 MPa, 1 h, 1000 °C (b) HIPed at 7 MPa, 1 h, 1000 °C and diepressed at 1000 °C to about 10% nominal strain.

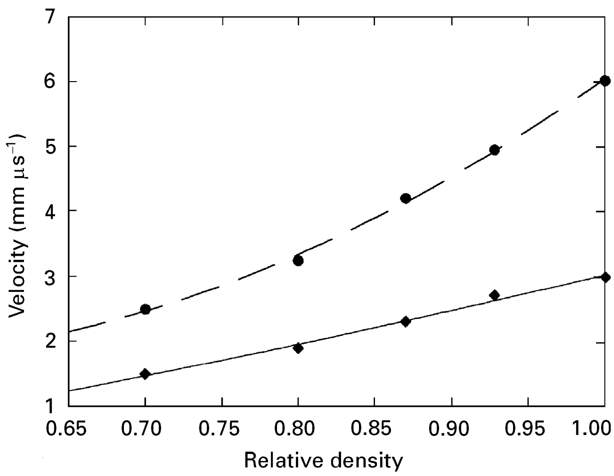

Figure 3 The, (●) longitudinal and (◆) shear velocities measured using ultrasonic transducer for specimens of different densities.

4. Results and discussion

Earlier work by Knudsen [6] has shown that below a critical density (or equivalently above a critical porosity) for a given packing, the properties of the powder compact go to zero. Since in the present work the green density is approximately 63% (i.e., the critical porosity is approximately 37%), it is assumed that the properties are zero at this density level. As the density increases during the HIPing cycle, the individual particles of the porous compact deform and form a bond with their neighbouring particles, hence the properties start to increase. However, at low density levels the increase is not significant.

In this work, the variation of the Young's modulus with relative density has been extensively studied. Fig. 4 shows the measured Young's modulus as a function of relative density. Wagh *et al.*, [13], postulated that the exponent 'n' in Equation 3 has a value of 2. Hasselman and Fulrath [14] found a similar exponent for glass with spherical pores. Ashby [15] gave an exact solution for E, as a power law of density for cellular solids and determined the exponent to be 2. However, Fig. 4 shows that the exponent n equal to 2 over predicts the effective Young's modulus for any density below 100%. Young's modulus data measured by different techniques by various researchers [16–18] have been found to be a linear function of the relative density. While the measured experimental data in the present work can perhaps be fitted to a linear correlation, a better fit is achieved by a power law of the form $E_p = E_0\rho^n$. Fig. 5 shows the variation of the measured shear modulus with relative density. Recently, Lam *et al.* [18] measured the Young's modulus of alumina, and proposed a linear model that correlates the relative Young's modulus to a density function given as $(\rho - \rho_0)\ /(1 - \rho_0)$, where ρ is the current relative density and ρ_0 is the initial relative density. Further, Lam *et al.* found that other mechanical properties such as the strain energy release rate as well as the stress intensity factor were also linear variations of the

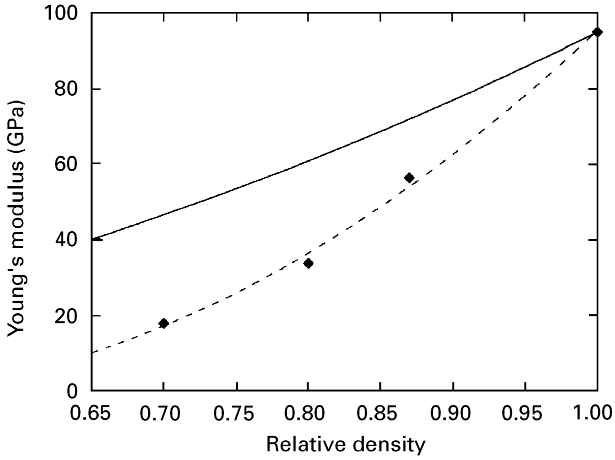

Figure 4 The, (◆) measured Young's modulus of alpha-two titanium aluminide compacts compared with power law models of the type $E = E_0 p^2$ (references [13–15]) signified with the solid line.

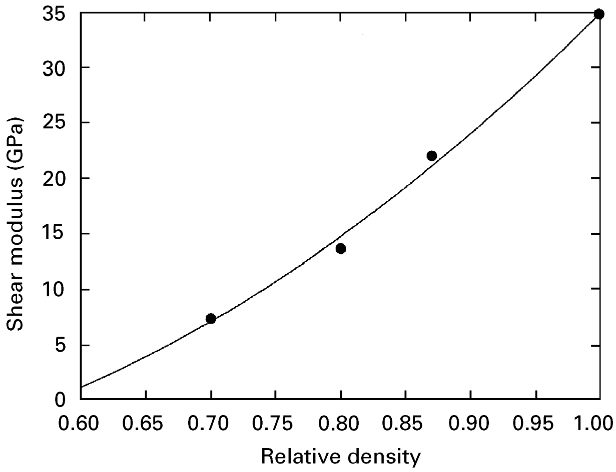

Figure 5 Measured shear modulus of alpha-two titanium aluminide compacts as a function of relative density.

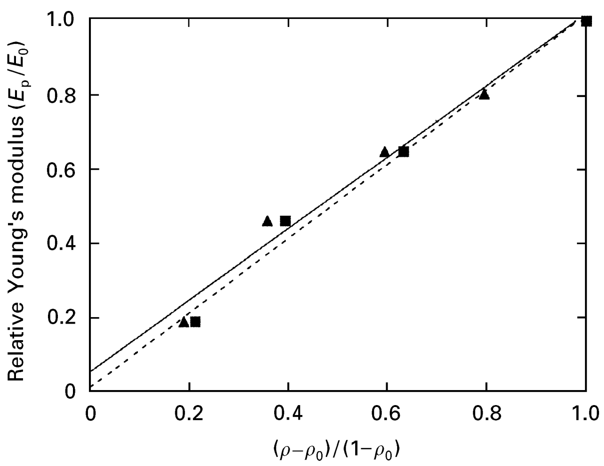

Figure 6 The; (▲) relative Young's and (■) relative shear Moduli (measured) plotted against the density function of Lam *et al.* [18] for different initial densities.

same density function. Fig. 6 shows the relative Young's modulus as well as shear modulus of alpha-two titanium aluminide plotted using the Lam *et al.* density function. Notably, Lam *et al.* observed a linear relation with a slope of 1 for alumina compacts of initial densities $\rho_0 = 0.50$ and 0.62.

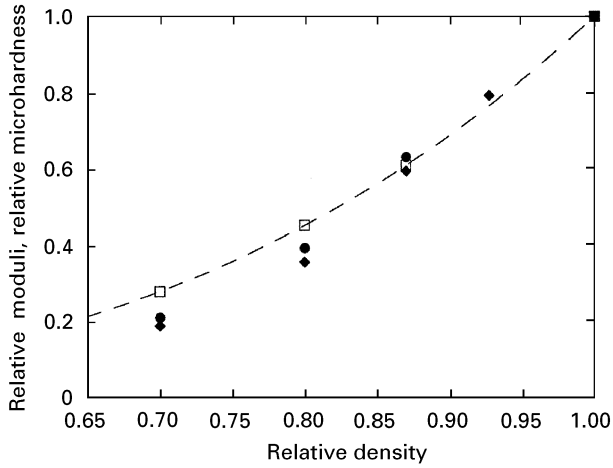

Figure 7 The; (◆) relative Young's and (●) relative shear moduli for alpha-two titanium compacts compared with (□) the relative microhardness from reference [19].

For alpha-two titanium aluminide also the linear relationship is seen to be valid with $\rho_0 = 0.63$, which is the measured tap density. However, the elastic modulus curve does not extrapolate exactly to zero modulus as in the case of alumina as noted in reference [18]. This is possibly due to the very low stiffness exhibited by the assembly of powders at tap density. It appears that at relative densities of the order of the tap density, there is a finite, albeit small, modulus which indicates that the assembly of powders has a small but finite load bearing capacity. Wang *et al.* [10], have shown that the ratio $E/E_0 = G/G_0$ where, G and G_0 are the shear moduli of partially dense and fully dense material respectively. Fig. 7 shows the relative Young's modulus and shear modulus plotted as a function of relative density. The two curves for E/E_0 and G/G_0 almost overlap in Figs. 6 and 7.

The relative microhardness of the alpha-two titanium aluminide compacts measured [19] is cross plotted in Fig. 7. It has been demonstrated by Shamasundar *et al.* [19] that the relative microhardness–relative density correlation can be used to derive the stress intensification factor. The latter has been used to model the consolidation of different ceramic and metal powders in a variety of processes [20, 21]. From Fig. 7, it is seen that the relative elastic and shear moduli follow the relative microhardness curve closely. Thus, the trends in Fig. 7 indicate that the same formulation could be used to model the consolidation as well as map the distribution of mechanical properties in the compact. Fig. 8 shows that the measured bulk modulus is more sensitive to the relative density. The dependence of bulk modulus on relative density is seen to be exponential in nature. An empirical relation of the form $K = 0.035 \exp(8.14\rho)$ seems to be the best fit for the experimental data. Fig. 9 shows the measured Poisson's ratio dependence on relative density. Ramakrishnan and Arunachalam proposed a model [5] for the effective Poisson's ratio in terms of porosity and the Poisson's ratio of the fully dense material (ν_0), which is of the form:

$$\nu = 0.25(4\nu_0 + 3p - 7\nu_0 p)/(1 + 2p - 3\nu_0 p) \quad (11)$$

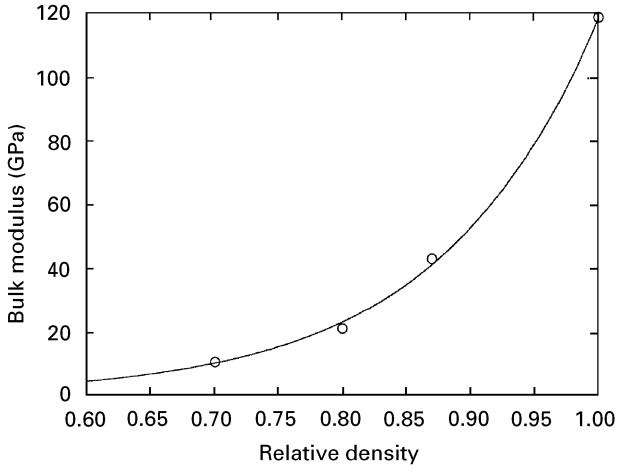

Figure 8 Measured bulk modulus of alpha-two titanium aluminide compacts as a function of relative density.

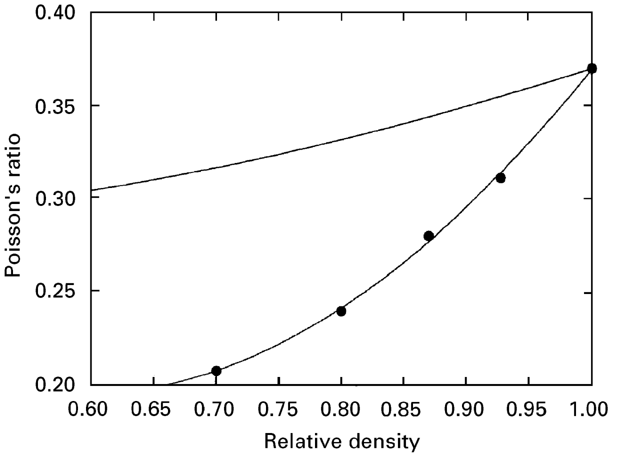

Figure 9 Measured Poisson's ratio of alpha-two titanium aluminide compacts of different relative densities. The measured data is symbolised by (•) and data calculated from the model of Ramakrishnan and Arunachalam is marked by the solid line.

They discussed the different trends of variation in Poisson's ratio with respect to relative density in the literature, i.e., an increase in the Poisson's ratio with porosity when the ν_0 was less than 0.25, a decrease in the Poisson's ratio with porosity when the ν_0 was more than 0.25, and a relative invariance of Poisson's ratio with porosity when the ν_0 was of the order of 0.25. It was further demonstrated using a finite element model that Equation 11 would predict experimental trends by different researchers for different materials. Fig. 9 shows that Equation 11 over predicts the experimental data by about 12%.

5. Conclusions

Ultrasonic techniques have been successfully used to measure the dynamic elastic moduli of porous compacts of a model material. By measuring the longitudinal and shear velocities as well as the density of a porous compact, it has been shown that the elastic moduli can be measured by properly accounting for the Poisson's ratio effect in the elastic half-space. It is seen that the elastic modulus and the shear modulus follow a power law relation as a function of relative density. With the choice of an appropriate form density function the relation can be presented with a linear model. The correlations of elastic and shear moduli with the relative density are found similar to the correlations of relative microhardness to density, indicating a possibility of mapping the elastic moduli, if the stress intensification factor of the material is known. The measured Poisson's ratio is seen to obey a linear correlation and the bulk modulus an exponential correlation with the relative density.

Acknowledgement

This research was performed at the Air Force Wright Laboratory, Materials Directorate, under AF Contracts: F33615-94-C-5213 (Drs. Matikas and Karpur), and F33615-92-C-5900 (Dr. Shamasundar). The authors gratefully acknowledge the technical support and guidance of Dr. S. L. Semiatin. S. Shamasundar gratefully acknowledges the support of the Materials Processing Technology Program during the course of this work.

References

1. W. H. DUCKWORTH, *J. Amer. Ceram. Soc.* **34** (1951) 1.
2. R. SPRIGGS, *ibid.* **44** (1961) 628.
3. J. K. MACKENZIE, *Proc. Phys. Soc.* **63B** (1950) 2.
4. K. K. PHANI and K. NIYOGI, *J. Mater. Sci. Lett* **6** (1987) 511.
5. N. RAMAKRISHNAN and V. S. ARUNACHALAM, *J. Mater. Sci.* **25** (1990) 3930.
6. F. P. KNUDSEN, *J. Amer. Ceram. Soc.* **42** (1959) 376.
7. R. W. RICE, in "Properties and Microstructure", edited by R. E. MacCrone, (Academic Press, New York, 1977).
8. G. W. MILTON, in "Physics and Chemistry of Porous Materials", Eds. D. L. Johnson and P. N. Sen, (American Institute of Physics, New York, 1984).
9. L. V. GIBIANSKY and S. TORQUATO, *Phys. Rev. Let.* **71** (1993) 2927.
10. J. C. WANG, *J. Mater. Sci.* **19** (1984) 801.
11. J. KRAUTKRAMER and H. KRAUTKRAMER, "Ultrasonic Testing of Materials", 4th Edn, (Springer Verlag, New York, 1990).
12. B. A. AULD, "Acoustic Fields and Waves in Solids", (Wiley, New York, 1973).
13. A. S. WAGH, R. B. POEPPEL and J. P. SINGH, *J. Mater. Sci.* **26** (1991) 3862.
14. D. P. H. HASSELMAN and R. M. FULRATH, *J. Amer. Ceram. Soc.* **47** (1965) 545.
15. M. F. ASHBY, *Metall. Trans A* **14A** (1983) 1755.
16. C. J. YU, R. J. HENRY, T. PRUCHER, S. PARTHASARATHI and J. JO, "Advances in P/M and Particulate Materials", Proceeding of the Powder Metallurgy World Congress sponsored by the Metal Powder Industries Federation and the American Powder Metallurgy Institute, **6** (1992) 319.
17. D. J. GREEN, C. NADER and R. BRENZY in "Sintering of Advanced Ceramics", 7, Eds. C. A. Handwerker, J. E. Blendell and W. Kaysser, (American Ceramics Society Westerville, OH 1990), p. 345.
18. D. C. LAM, F. F. LANGE and A. G. EVANS, *J. Amer. Ceram. Soc.* **77** (1994) 2113.
19. S. SHAMASUNDAR, R. E. DUTTON and S. L. SEMIATIN, *Scripta Metall Mater.* **31** (1994) 521.
20. *idem., J. Mater. Proc. Tech.* **48** (1994) 817.
21. R. E. DUTTON, S. SHAMASUNDAR and S. L. SEMIATIN, *Metal. Mater. Trans. A* **26A** (1995) 2041.

Received 21 March 1995
and accepted 18 March 1996